

Dean Doty's "minimal sufficiency" argument was presented as an enthymeme. For those who are unfamiliar with logic, his reasoning is offered in detail hereinafter.

4.1 Minimal Sufficiency

A given problem can admit many sufficient statistics.

4.1.1 Example

Consider the Normal location model $X_i \sim N(\theta, 1)$. The data itself, (X_1, \dots, X_n) is sufficient. Also, a re-ordering $(X_{(1)}, \dots, X_{(n)})$ is sufficient. We will show that the mean $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ is minimally sufficient.

4.1.2 Definition

A statistic T is "Minimally sufficient" if T is sufficient and for all other sufficient statistics U there exists some function f such that $T = f(U)$, a.s. $\mathbb{P}_\theta, \forall \theta \in \Theta$

We want simpler criteria to recognize minimal sufficiency.

Theorem 4.1. (a) Let $\mathcal{P} = \{\mathbb{P}_\theta | \theta_0, \dots, \theta_k\}$ be a regular family with finitely many members. Then $T(X) = \left(\frac{p(x; \theta_1)}{p(x; \theta_0)}, \dots, \frac{p(x; \theta_k)}{p(x; \theta_0)} \right)$ is minimal sufficient.

(b) Given nested families $\tilde{\mathcal{P}} \subseteq \mathcal{P}$ such that $(\tilde{\mathcal{P}} \text{ a.e.})$ implies $(\mathcal{P} \text{ a.e.})$, if T is minimal sufficient for $\tilde{\mathcal{P}}$ and sufficient for \mathcal{P} then T is minimal sufficient for \mathcal{P} .

Therefore, Lindley and DePree's "grievance does not meet \mathbf{T} , the minimal sufficiency for initiating a faculty grievance."

We have borrowed from Wainwright's Theoretical Statistics Lecture 4—September 7, 2006. See, http://www.stat.berkeley.edu/~gvrocha/STAT210A/scribing/lectures/stat210a_lecture_04.pdf